# APPENDIX F
# ACTIVITY #1 - CALCULATION OF THE TILT OF THE EARTH'S AXIS AND THE OBSERVER'S LATITUDE

The calculation of the Tilt of the Earth's Axis and the Observer's Latitude is essentially a reversal of the calculation sequence shown for the construction of the enclosure (Appendix A). In the discussion below, "tip of the gnomon" can usually be read in lieu of "opening of the enclosure" for participants who use a gnomon to cast a shadow instead of an enclosure with an opening to pass a light ray.

**Step 1:** setup a coordinate system with:

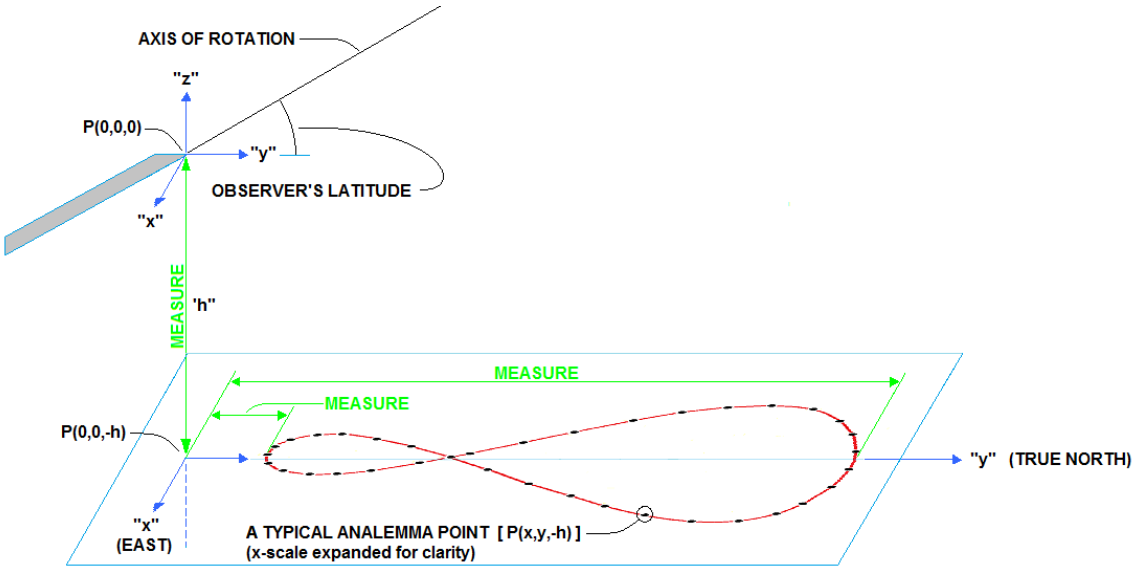
- P(0,0,0) at the opening of the enclosure / tip of the gnomon.
- the x-axis as east / west (positive being eastward).
- the y-axis as north / south (positive being northward).
- the z-axis as up / down (positive being upward).

Note that the analemma is in the x-/y-plane at $z = -h$.

**Step 2:** Locate the point both directly below the opening in the enclosure and in the plane containing the analemma … this point will be referred to as "P(0,0,-h)." For the construction described in Appendix A, a heavy sewing needle works well when suspended through the opening by a thread on a windless day with the enclosure in its observing position.

**Step 3:** Make the following three measurements from P(0,0,-h) (determined in Step-1 above):

- Distance from the opening in the enclosure to the plane in which the analemma lies; in Appendix A, this distance is referred to as "h"
- Distance to the Summer Solstice point. This is the point on the analemma curve closest to the x-axis.
- Distance to the Winter Solstice point. This is the point on the analemma curve farthest from the x-axis.




**Step 4:** Using the measurements above, calculate the altitude of the Sun at both Summer and Winter Solstices:

(AltSummerSolstice) = arctan(h / (DistanceToSummerSolsticePoint) )

(AltWinterSolstice) = arctan(h / (DistanceToWinterSolsticePoint) )

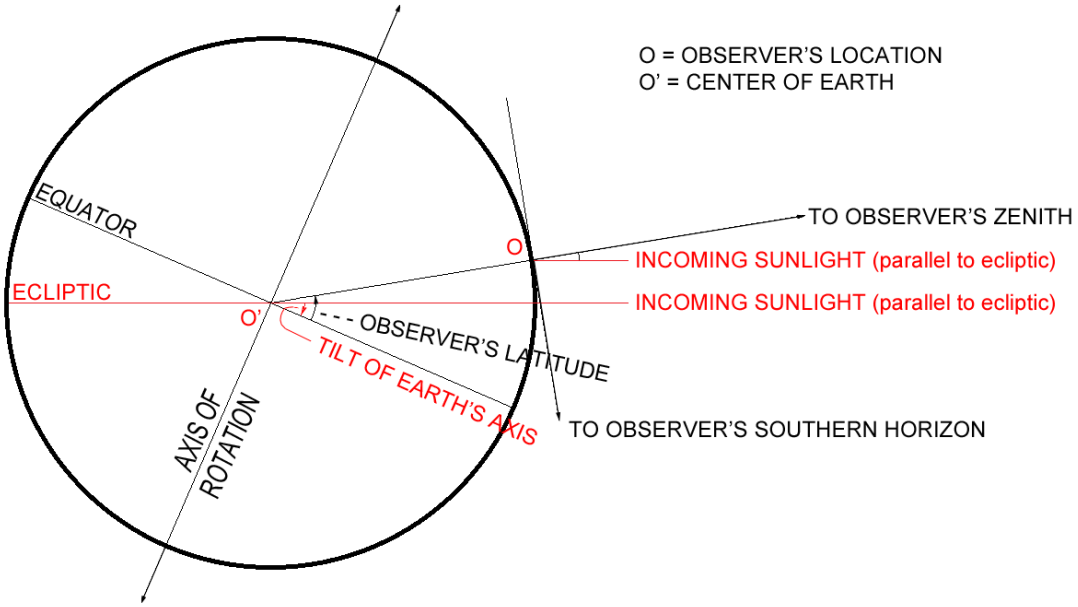
**Step 5:** Specify the relationships between the altitude of the Sun at Summer Solstice, the tilt of the Earth's axis, and the Observer's Latitude. With reference to the sketch below, specifically the 90° angle between the Observer's southern horizon and the Observer's zenith:

(AltSummerSolstice) + (ObsLatitude) – (TiltOfAxis) = 90°

The altitude of the Sun at Summer Solstice (AltSummerSolstice) is calculated from the dimensions of the analemma and the observing apparatus (Step 3).

Rearranging the equation gives:

(ObsLatitude) – (TiltOfAxis) = 90° - (AltSummerSolstice)



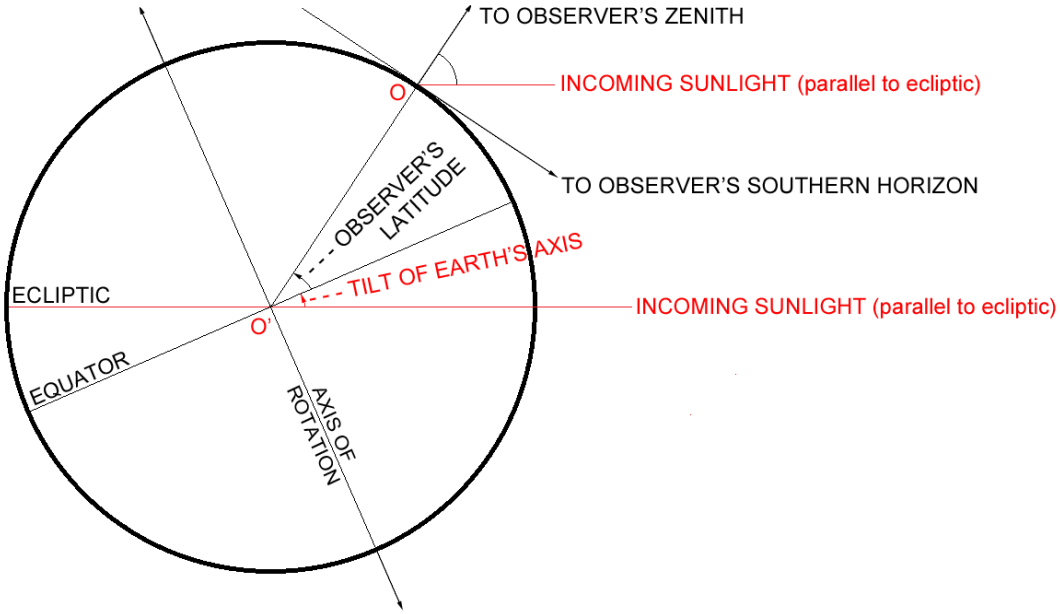

**<u>Step 6</u>:** Specify the relationships between the altitude of the Sun at Winter Solstice, the tilt of the Earth's axis, and the Observer's Latitude. With reference to the sketch below, specifically the 90° angle between the Observer's southern horizon and the Observer's zenith:

$$(\text{AltWinterSolstice}) + (\text{ObsLatitude}) + (\text{TiltOfAxis}) = 90°$$

The altitude of the Sun at Winter Solstice (AltWinterSolstice) is calculated from the dimensions of the analemma and the observing apparatus (Step 3).

Rearranging the equation gives:

$$(\text{ObsLatitude}) + (\text{TiltOfAxis}) = 90° - (\text{AltWinterSolstice})$$




**Step 7:** Specify the Observing Latitude by adding the equations from Step 4 and Step 5, above:

(ObsLatitude) – (TiltOfAxis) + (ObsLatitude) + (TiltOfAxis) =
90° - (AltSummerSolstice) + 90° - (AltWinterSolstice)

Rearranging the equation gives:

2*(ObsLatitude) = 2*90° - (AltSummerSolstice) - (AltWinterSolstice)

Again, rearranging the equation gives:

(ObsLatitude) = 90° - ( (AltSummerSolstice) + (AltWinterSolstice) ) / 2

Use the solar altitudes calculated in Step 3, above.

**Step 8:** Specify the Tilt of the Earth's Axis by subtracting the equation from Step 4, above, from the equation from Step 5, above:

(ObsLatitude) + (TiltOfAxis) – (ObsLatitude) + (TiltOfAxis) =
90° - (AltWinterSolstice) – 90° + (AltSummerSolstice)

Rearranging the equation gives:

2*(TiltOfAxis) = (AltSummerSolstice) – (AltWinterSolstice)

Again, rearranging the equation gives:

(TiltOfAxis) = ( (AltSummerSolstice) – (AltWinterSolstice) ) / 2

Use the solar altitudes calculated in Step 3, above.

## Additional Links